Access to Science, Engineering and Agriculture: Mathematics 1
MATH00030
Chapter 1 Exercises

1. Without using a calculator, calculate the following.

   (a) $\frac{3}{7} - \frac{3}{8}$.

   (b) $\frac{3}{5} + \frac{1}{9}$.

   (c) $3 - \frac{10}{11}$.

   (d) $\frac{1}{2} + \frac{2}{3} + \frac{4}{5}$.

2. Without using a calculator, calculate the following.

   (a) $\frac{2}{3} \times \frac{4}{7}$.

   (b) $\frac{8}{7} \times \left(-\frac{1}{3}\right)$.

   (c) $-\frac{5}{4} \times \left(-\frac{4}{5}\right)$.

   (d) $4 \times \left(-\frac{1}{4}\right)$.

   (e) $\frac{2}{3} \div \frac{1}{3}$.

   (f) $-\frac{6}{7} \div \left(-\frac{7}{6}\right)$.

   (g) $\frac{10}{3} \div \left(-\frac{2}{7}\right)$.

   (h) $0 \div 1$.

   (i) $1 \div 0$.

3. Without using a calculator, calculate the following.

   (a) $6 \div 7 \times 8 + 9$.

   (b) $6 \div 7 \times (8 + 9)$.

   (c) $6 \div (7 \times 8 + 9)$.

   (d) $6 \div (7 \times 8) + 9$.

4. Without using a calculator, calculate the following.

   (a) 3^3.

   (b) $(-2)^5$.

   (c) $\left(\frac{1}{2}\right)^4$.

   (d) $\sqrt{16}$.

   (e) $\sqrt[3]{64}$.

   (f) $\sqrt[15]{1}$.

   (g) $(64)^{\frac{2}{3}}$.

   (h) $(16)^{-\frac{3}{2}}$.

   (i) $\left(\frac{4}{25}\right)^{\frac{3}{2}}$.

   (j) $\left(\frac{27}{8}\right)^{-\frac{5}{3}}$.

5. Simplify the following expressions by expressing them as a single power of x.

   (a) $x^9 \times x^6$.

   (b) $x^{10} \times x^{-13}$.

   (c) $x^{\frac{3}{4}} \times x^{\frac{1}{3}}$.

   (d) $x^{\frac{1}{2}} \times x^{-\frac{3}{4}}$.

   (e) $(x^3)^4$.

   (f) $(x^{-3})^2$.

   (g) $\left(x^{\frac{1}{2}}\right)^{-\frac{1}{3}}$.

   (h) $x^6 \div x^4$.

   (i) $x^{\frac{1}{2}} \div x^{-\frac{2}{3}}$.

   (j) $\left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}\right)^{\frac{3}{2}}$.

6. Without using a calculator, calculate the following.

   (a) 5×4^2.

   (b) $(5 \times 4)^2$.

   (c) $5 \div 2^3 + 4$.

   (d) $5 \div (2^3 + 4)$.

   (e) $(5 \div 2)^3 + 4$.

   (f) $3 \times 4 \div 5 + 2^2$.

   (g) $3 \times 4 \div (5 + 2)^2$.

(h) $3 \times (4 \div 5 + 2)^2$.

(i) $(3 \times 4 \div 5 + 2)^2$.

(j) $3 \times (4 \div 5 + 2^2)$.

7. Simplify the following expressions by expressing them as a power of x, y and/or z, as appropriate.

(a) $\left(x^2 \sqrt[3]{y}\right)^3$.

(b) $\left(x^{-3} y^{\frac{1}{2}}\right)^{\frac{2}{3}}$.

(c) $\left(x^{-4} y^{-\frac{2}{3}}\right)^{-2}$.

(d) $\left(x y^{-\frac{1}{3}} z^{\frac{1}{2}}\right)^6$.

8. Without using a calculator, find the following logarithms.

(a) $\log_4 16$.

(b) $\log_5 125$.

(c) $\log_{36} 6$.

(d) $\log_{20} \dfrac{1}{20}$.

(e) $\log_8 \dfrac{1}{64}$.

(f) $\log_{27} \dfrac{1}{3}$.

9. Express the following in terms of $\log_a x$ and $\log_a y$.

(a) $\log_a \left(x^4 y^{\frac{1}{2}}\right)$.

(b) $\log_a \left(\left(\dfrac{x^2}{y^3}\right)^{-2}\right)$.

(c) $\log_a \left(x^{\log_a (y^2)}\right)$.

10. Perform the following approximations.

(a) Approximate 15.450 to one decimal place.

(b) Approximate 9.95 to one decimal place.

(c) Approximate 0.004 to two decimal places.

(d) Approximate 10 to three decimal places.

(e) Approximate -1.56 to one decimal place.

(f) Approximate -10.655 to two decimal places.

11. Perform the following approximations.

    (a) Approximate 7595462381 to three significant figures.
    (b) Approximate 0.000125 to two significant figures.
    (c) Approximate 29.95 to two significant figures.
    (d) Approximate 30 to four significant figures.
    (e) Approximate -1.45 to two significant figures.
    (f) Approximate -0.01216 to three significant figures.

12. Convert the following to scientific notation.

    (a) Express 14674.45 in scientific notation.
    (b) Express 0.00436 in scientific notation.
    (c) Express 43543.4445 in scientific notation to three significant figures.
    (d) Express 0.00345 in scientific notation to four significant figures.

13. Simplify the following expressions.

    (a) $(2x^3 - 2x^2 + 3x - 4) + (-x^3 + 3x + 4)$.
    (b) $(-3x^3 - 5x + 7) - (-4x^3 + 3x^2 - 3x + 9)$.
    (c) $(3x^7 + 3x^3 - 2x^{-1} + 4x^{-4}) + (7x^3 + 7 - x^{-1} - 3x^{-4})$.

14. Multiply out the following expressions.

    (a) $3x^3(x^2 - 3x + 3)$.
    (b) $(x^2 + 3x)(-3x^2 + 5)$.
    (c) $(4x - 2)(x^2 + 4x + 1)$.
    (d) $(4x^2 - x + 1)(-x^2 - x - 1)$.
    (e) $(-x^{-1} - 2x^{-2})(x^{-1} + 3x^{-2})$.

15. Perform long division on each of the following, giving the quotients and remainders.

    (a) $63211 \div 6$.
    (b) $324563 \div 5$.
    (c) $573653 \div 23$.
    (d) $46375835 \div 521$.

16. Perform long division on each of the following, giving the quotients and remainders.

    (a) $\dfrac{x^2 - x + 1}{x + 1}$.
    (b) $\dfrac{x^3 - x^2 + 2x + 2}{x - 1}$.
    (c) $\dfrac{3x^3 - 5x^2 + x - 2}{3x + 1}$.
    (d) $\dfrac{2x^4 - 5x^2 + x - 2}{x^2 + x + 1}$.

17. Evaluate the following.

(a) $\sum_{i=1}^{5} i$.

(b) $\sum_{i=0}^{4} i^3$.

(c) $\sum_{i=-2}^{2} 2i^2$.

18. Expand the following.

(a) $\sum_{i=-2}^{1} x^i$.

(b) $\sum_{i=0}^{4} x^{2i}$.

(c) $\sum_{i=-2}^{2} ix^3$.

19. Calculate the following binomial coefficients without using a calculator.

(a) $\binom{11}{2}$.

(b) $\binom{20}{3}$.

(c) $\binom{88}{86}$.

(d) $\binom{100}{100}$.

20. Expand the following using The Binomial Theorem.

(a) $(x + y^2)^2$.

(b) $(2x + 3y)^2$.

(c) $(2 + 3y)^3$.

(d) $(3x + y^3)^3$.